# Bifröst

Heimdall is planning to make his yearly journey along *Bifröst*, the rainbow path to the heavens. The path consists of k cities, and consecutive cities in the path are connected by *bridges*. Each bridge is a certain colour, denoted by a non-negative integer between 0 and 5 000 inclusive. The k-1 bridges are numbered 1 to k-1 from left to right and the *i*th bridge is coloured  $c_i$ .

Heimdall's journey starts in the leftmost city and ends in rightmost city, possibly revisiting some cities more than once. In total, he crossed bridges A times. The *i*th bridge he crossed was coloured  $a_i$ , and he wrote this down in his notebook.

For example, if there were k = 8 cities and the bridges were coloured 1, 3, 2, 3, 4, 4, 5, then the colours written in his notebook could be:

- 1, 3, 2, 3, 4, 4, 5.
- 1, 3, 3, 1, 1, 3, 2, 3, 4, 4, 5.
- $\bullet \ \ 1,3,2,3,4,4,4,4,3,3,4,4,5,5,5.$

These are shown in the figure below.



Figure 1: Three possible journeys. The colour of each bridge is written above it.

This year, Loki also made a journey from the leftmost city to the rightmost city. In total, he crossed bridges B times, the *i*th one coloured  $b_i$ . He wrote this down in his notebook too.

Heimdall thinks Loki might be lying, so he has asked you to determine if there is any integer k and sequence  $c_1, c_2, \ldots, c_{k-1}$  that agrees with both his and Loki's notebooks.

### Subtasks and Constraints

For all subtasks:

- $1 \le A, B \le 5\,000.$
- $0 \le a_i, b_i \le 5\,000$  for all *i*.

Additional constraints for each subtask are given below.

Subtask	Points	Additional constraints
1	6	$a_i = 0$ and $b_i = 0$ for all $i$ .
2	10	Each colour appears at most once in Heimdall's notebook.
3	16	Each colour (except 0) appears at most once in Heimdall's notebook.
4	17	$A, B \leq 15$
5	34	$A, B \leq 100$
6	17	$A, B \le 5000$

## Input

- The first line of input contains the two integers A and B.
- The second line contains A integers  $a_1, a_2, \ldots, a_A$ .
- The third line contains B integers  $b_1, b_2, \ldots, b_B$ .

# Output

If there is no sequence that agrees with both Heimdall and Loki's notebooks, output the single line IMPOSSIBLE.

Otherwise, output two lines:

- On the first line, print k, the number of cities. Your choice of k must be at most 5001.
- On the second line, print the k-1 integers  $c_1, c_2, \ldots, c_{k-1}$ .

If there are multiple correct sequences, you can print any of them. You **do not** need to minimize nor maximize k.

Sample Input 1	Sample Output 1
11 15	8
1 3 3 1 1 3 2 3 4 4 5	1 3 2 3 4 4 5
1 3 2 3 4 4 4 4 3 3 4 4 5 5 5	

Sample Input 2	Sample Output 2
4 3 5 0 0 3 5 0 3	IMPOSSIBLE

# Explanation

Sample Input 1 corresponds to the case shown in Figure 1. One possible solution has k = 8 cities, with the k - 1 bridges coloured 1, 3, 2, 3, 4, 4, 5 from left to right. Then, Heimdall could have taken the following journey:

- He began by walking right along bridge 1 (with colour 1),
- He then walked right along bridge 2 (with colour 3),
- He then walked left along bridge 2 (with colour 3),
- He then walked left along bridge 1 (with colour 1),
- He then walked right along bridge 1 (with colour 1),
- He then walked right along bridge 2 (with colour 3),
- He then walked right along bridge 3 (with colour 2),
- He then walked right along bridge 4 (with colour 3),
- He then walked right along bridge 5 (with colour 4),
- He then walked right along bridge 6 (with colour 4),
- He then walked right along bridge 7 (with colour 5), finishing his journey.

Loki could have taken the following journey:

- He began by walking right along bridge 1 (with colour 1),
- He then walked right along bridge 2 (with colour 3),
- He then walked right along bridge 3 (with colour 2),
- He then walked right along bridge 4 (with colour 3),
- He then walked right along bridge 5 (with colour 4),
- He then walked right along bridge 6 (with colour 4),
- He then walked left along bridge 6 (with colour 4),
- He then walked left along bridge 5 (with colour 4),
- He then walked left along bridge 4 (with colour 3),
- He then walked right along bridge 4 (with colour 3),
- He then walked right along bridge 5 (with colour 4),
- He then walked right along bridge 6 (with colour 4),
- He then walked right along bridge 7 (with colour 5),
- He then walked left along bridge 7 (with colour 5),
- He then walked right along bridge 7 (with colour 5), finishing his journey.

In Sample Input 2, there is no sequence that agrees with both Heimdall and Loki's notebooks.

# Bongi

Stop what you are doing, a new very fun game has just been released: *Bongi*. The game is played using N balls. The *i*th ball has the positive integer  $a_i$  written on it.

You must divide the balls into K baskets, numbered 1 to K. Every ball must go into one of the baskets and every basket must contain at least one ball. The *value* of a basket is the sum of the integers on the balls in it. Your *score* is the geometric mean<sup>1</sup> of the values of the baskets.

Consider the following example with K = 2 baskets and N = 5 balls of value 12, 14, 16, 17 and 25. One possible solution is as follows:

- The first basket contains 16 and 17, and
- The second basket contains 25, 12 and 14.

The score of this solution is  $(16 + 17)^{0.5} \cdot (25 + 12 + 14)^{0.5} \approx 41.02$ .

You do not have to produce the maximum score possible, instead you are awarded points based on the score you are able to achieve.

This is an **output only** problem. You do not submit source code for this task. Instead, you are given a series of input files and must submit the corresponding output files.

### Constraints

For all test cases:

- $1 \le a_i \le 10000$  for all *i*.
- $2 \le K \le N \le 100$ .

#### Input

- The first line of input contains the two integers N and K.
- The second line contains N integers  $a_1, a_2, \ldots, a_N$ .

### Output

Output K lines, each containing a collection of integers, such that the *i*th line contains the values of the balls in the *i*th basket.

<sup>&</sup>lt;sup>1</sup>The geometric mean of k values  $a_1, a_2, ..., a_k$  is defined as  $a_1^{1/k} \times a_2^{1/k} \times ... \times a_k^{1/k}$ . For example, the geometric mean of 8, 9 and 10 is  $8^{1/3} \times 9^{1/3} \times 10^{1/3} \approx 2 \times 2.080 \times 2.154 \approx 8.963$ .

## Scoring

If you do not put every ball into exactly one basket, and have each basket contain at least one ball, you will score 0% of the points for that test case.

Otherwise, let X be the score of your solution, B be the judges' best solution and  $S = a_1 + a_2 + \cdots + a_N$ . Then you will score

$$\min\left(100 \cdot \frac{10 + S/K - B}{10 + S/K - X}, 100\right)$$

percent of the points for that test case.

Note that it is possible to prove that  $X \leq S/K$ , thus the scoring function is well-defined.

Testcase	Points	N	K	Judges' best score
1	5	5	2	42
2	10	20	10	9564.453334019241
3	10	100	10	55237.69999809915
4	15	100	20	25324.1999731493
5	20	100	30	16980.161334096203
6	20	100	40	13886.860798648839
7	10	100	50	10086.367502580433
8	10	100	60	8392.505093895417

### Submitting

To submit your output file for test case X, you must upload a file named output\_X.txt. For example, for test case 4, the output file must be named output\_4.txt.

You can choose to upload your output file for each test case individually, or together in a zip file.

Your score for this problem is the sum of your maximum scores for each test case, over all submissions you have made to this problem.

Bongi

Sample Input 1	Sample Output 1
5 2	16 17
16 25 17 14 12	25 12 14

## Explanation

The score of the solution is  $X = (16 + 17)^{0.5} \cdot (25 + 12 + 14)^{0.5} \approx 41.02$ . For this example, the best solution of the jury has score 42. Therefore, the percentage of points obtained by this output is

$$\approx 100 \cdot \frac{10 + 84/2 - 42}{10 + 84/2 - 41.02} \approx 91.11.$$

This example corresponds to the first test, which is worth 5 points. The number of points obtained when submitting this solution is 4.56/5.

## Sample Code

You are provided with two files, template.cpp and bongi.hpp, to help you implement your solution.

The file **bongi.hpp** contains two functions that you may find useful:

```
double get_score(vector<vector<int> > solution);
```

This function returns the score of a solution. For example, if solution = [[ 16, 17 ], [ 25, 12, 14 ]], then get\_score(solution) returns (approximately) 41.02.

```
double get_percentage(vector<vector<int> > solution);
```

This function returns the percentage of points that a solution would be given. For example, if solution = [[ 16, 17 ], [ 25, 12, 14 ]], then get\_percentage(solution) returns (approximately) 91.11.

The file template.cpp contains an example usage of get\_percentage. You can compile template.cpp using this command:

```
g++ -Wall -std=c++17 -O2 -o template template.cpp
```

# Hedge Maze

Puss and Kitty must face the diabolical challenge of the *Down-Right Hedge Maze*. The maze is an  $R \times C$  grid of cells with R rows (numbered 1 to R from top to bottom) and C columns (numbered 1 to C from left to right). The cell in the rth column and cth row is denoted (r, c). There are three types of cells, denoted by an uppercase character:

- D: It is only allowed to move downwards from this cell (if there is a cell there).
- R: It is only allowed to move rightwards from this cell (if there is a cell there).
- B: It is allowed to move downwards or rightwards from this cell (if there is a cell there).

It is possible to reach cell (R, C) from every cell in the maze.

D	В	В	R	D	D	В	D
R	D	D	R	R	R	R	В
В	R	D	В	В	D	В	В
В	D	D	D	R	R	D	D
R	В	В	R	R	В	В	D

Figure 1: The maze from Sample Input 1. Thick black lines are drawn between cells where a move is disallowed.

To defeat the challenge, Puss and Kitty must answer Q queries. In each query, you are given four integers  $a_i, b_i, c_i$  and  $d_i$  and must answer the following question: If Puss starts in the cell  $(a_i, b_i)$  and Kitty starts in the cell  $(c_i, d_i)$ , what is the the fewest total moves they must make to meet at a common cell? Puss and Kitty start in different cells.

#### Subtasks and Constraints

For all subtasks:

- $2 \le R \le 1000$
- $2 \le C \le 500\,000$
- $4 \le R \times C \le 1\,000\,000$
- $1 \le Q \le 100\,000$
- $1 \le a_i \le R$  and  $1 \le b_i \le C$  for all i.
- $1 \le c_i \le R$  and  $1 \le d_i \le C$  for all i.
- $(a_i, b_i) \neq (c_i, d_i)$  for all *i*.

Additional constraints for each subtask are given below.

Subtask	Points	Additional constraints
1	9	$R, C \le 20$
2	17	$(a_i, b_i) = (1, 1)$ for all <i>i</i> .
3	20	Each cell will be type D or R.
4	29	There is a sequence of moves from $(1,1)$ to any cell in the maze.
5	25	No additional constraints.

# Input

- The first line of input contains the three integers R, C, Q.
- R lines follow, each containing a string of C characters, describing the maze. The cth character on the rth line represents the type of the cell (r, c).
- Q lines follow, describing the queries. The *i*th line contains the four integers  $a_i$ ,  $b_i$ ,  $c_i$  and  $d_i$ .

## Output

Output Q lines. On the *i*th line, print the answer to the *i*th query.

Sample Input 1	Sample Output 1
583	9
DBBRDDBD	4
RDDRRRB	7
BRDBBDBB	
BDDDRRDD	
RBBRRBBD	
1 2 3 7	
3 3 1 1	
3 6 4 4	

Sample Input 2	Sample Output 2
4 4 4	7
BBRD	4
DRBB	4
BDRD	2
BBRB	
2 1 3 3	
1 3 2 2	
3 3 1 3	
4 1 3 2	

# Explanation

In Sample Input 1:

- For the first query, Puss and Kitty start in cell (1, 2) and (3, 7) respectively. They can meet in cell (3, 8), requiring 8 and 1 moves respectively, for a total of 9 moves.
- For the second query, Puss and Kitty start in cell (3,3) and (1,1) respectively. They can meet in cell (3,3), requiring 0 and 4 moves respectively, for a total of 4 moves.
- For the third query, Puss and Kitty start in cell (3,6) and (4,4) respectively. They can meet at cell (5,7), requiring 3 and 4 moves respectively, for a total of 7 moves.



Figure 2: Sample Input 1

In Sample Input 2:

- For the first query, Puss and Kitty start in cell (2, 1) and (3, 3) respectively. They can meet in cell (4, 4), requiring 5 and 2 moves respectively, for a total of 7 moves.
- For the second query, Puss and Kitty start in cell (1,3) and (2,2) respectively. They can meet in cell (2,4), requiring 2 and 2 moves respectively, for a total of 4 moves.
- For the third query, Puss and Kitty start in cell (3,3) and (1,3) respectively. They can meet at cell (3,4), requiring 1 and 3 moves respectively, for a total of 4 moves.
- For the fourth query, Puss and Kitty start in cell (4, 1) and (3, 2) respectively. They can meet at cell (4, 2), requiring 1 and 1 move respectively, for a total of 2 moves.



Figure 3: Sample Input 2. Queries 3 and 4 are shown on a separate copy of the maze for clarity.