French-Australian Regional Informatics Olympiad

Thursday 9th March, 2017

Duration: 4 hours

3 questions

All questions should be attempted

Problem 1
Pair Programming

Input File: *standard input*
Output File: *standard output*

Time and Memory Limits: 3 seconds, 256 MB

Students are yelling at each other. Hiding under tables. Standing on top of tables. Re-enacting lightsaber duels with pens. Swinging on the ceiling fans... Walking in you are horrified to find your software class wreaking havoc and in a shambles. Shoulders hunched and palm pressed firmly against your face, you attempt to restore some semblance of order by asking some students to program in pairs.

There are N students in your class. Each student writes in a particular programming language, and uses a particular text editor. You've noticed that a pair of students is *cooperative* if they write in the same language or use the same editor, or both.

You would like to write a program that finds the largest number of cooperative pairs you can form, where no student is present in more than one pair. Your program should also output a valid set of pairings (see *Output* and *Scoring* sections below for further details).

Input

The first line of input will contain a single integer N: the number of students in your class. Students are numbered from 1 to N.

N lines follow, the i-th containing two integers l_i e_i: the language and editor that student i uses. Languages and editors are denoted by integers between 1 and N, inclusive.

Output

The first line of output must contain a single integer A: the maximum number of cooperative pairs that you can form.

A lines must follow. Each of these lines must contain two integers identifying two students that should be paired together. No student's number may appear more than once among these A lines. The order of these lines and the order of students within each line does not matter. If there is more than one largest set of pairings, any valid one will be accepted.

See the *Scoring* section below for further details.

Sample Input

```
10
3 6
8 2
4 4
4 7
5 2
5 4
2 1
7 4
9 10
9 10
```

Sample Output

```
4
2 5
8 6
3 4
9 10
```

Explanation

We can pair together

- Student 2 and student 5 (both use editor 2);
- Student 8 and student 6 (both use editor 4);
- Student 3 and student 4 (both write in language 4); and
- Student 9 and student 10 (both write in language 9 and both use editor 10)

for a total of 4 pairs, which is the most we can form in this case. Note that some languages and some editors are not used by any student.

Subtasks & Constraints

For all subtasks, $1 \leq N \leq 1\,000\,000$ and for all $1 \leq i \leq N$, $1 \leq l_i, e_i \leq N$.

- For Subtask 1 (20 points), the language a student writes in has the same number as the editor they use. Specifically, $l_i = e_i$ for all i.
- For Subtask 2 (20 points), $N \leq 50$.
- For Subtask 3 (20 points), $N \leq 1\,000$.
- For Subtask 4 (40 points), no additional constraints apply.

Scoring

For each test case, you will score 100% if the integer A on the first line of output is correct and you provide a correct set of A cooperative pairings. Otherwise, if the integer A is correct, you will score 40% for that test case *even if*:

- You do not supply a set of pairings; or
- You attempt to provide a set of pairings, but it is not a valid set of A cooperative pairs.

Otherwise, you will score 0% for that test case.

Your score for each subtask will be the **minimum** score among all testcases in that subtask. Your score for this problem will be the **maximum** score among all submissions you have made to this problem.

Problem 2
Pyramid Cake

Input File: *standard input*
Output File: *standard output*

Time and Memory Limits: 4 seconds, 256 MB

It's your birthday! For your Egyptian themed party, you decide to order a layered pyramid cake from one of Melbourne's famous hipster cafés. Following the hipster trend, this café packages cakes in an absurdly impractical but uniquely shaped box. The base of the box is a rectangular grid consisting of R rows and C columns, with rows numbered 1 to R from top to bottom and columns numbered 1 to C from left to right. However, the height of the box varies from cell to cell over the grid. To prevent the cake being squashed by the box, the number of layers at any point cannot exceed the height of the box at that point.

A pyramid cake consists of one or more rectangular layers, aligned with the sides of the box. Each layer has its top-left corner anchored at the top-left corner of the box—it must always cover cell $(1, 1)$. To maintain structural integrity, each layer must rest completely on top of the one below, so that each cell covered by a particular layer is also covered by every layer underneath.

The volume of a layer cake is the sum of the number of cells covered by each layer. Wanting to eat as much delicious cake as possible, you would like to write a program to determine the maximum volume of a pyramid cake you can order. If no valid cakes fit inside the box, output 0.

Input

The first line of input will contain two integers R and C, describing the number of rows and columns in the box, respectively.

R lines follow, each containing C integers. The jth number in the ith line, H_{ij}, describes the maximum number of cake layers allowed at row i and column j of the grid.

Output

Your program must output a single integer: the maximum volume of a pyramid cake that fits inside the box. Since this number may be large, you may wish to use 64-bit integer types (such as `long long` in C/++) in your program.

Sample Input 1

```
4 4
1 1 1 0
1 1 0 1
1 1 1 0
1 0 0 1
```

Sample Output 1

```
6
```

Sample Input 2

```
4 5
5 4 9 3 3
4 3 5 6 0
2 2 1 1 4
2 1 3 5 8
```

Sample Output 2

```
36
```

Explanation

In the first sample case, you can order a cake with a single layer with its top-left at (1, 1) and its bottom-right at (3, 2). This gives you a total volume of 6, which is the maximum among all cakes you can order for this case.

In the second sample case, we can order the cake described below.

5	4	9	3	3
4	3	5	6	0
2	2	1	1	4
2	1	3	5	8

Layer 1

5	4	9	3	3
4	3	5	6	0
2	2	1	1	4
2	1	3	5	8

Layer 2

5	4	9	3	3
4	3	5	6	0
2	2	1	1	4
2	1	3	5	8

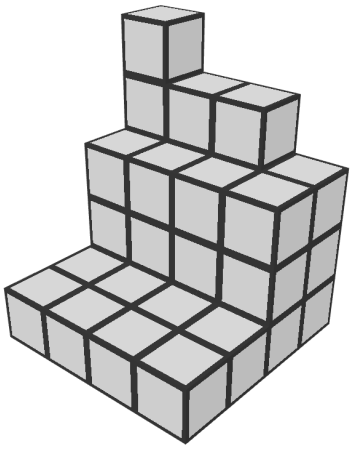
Layer 3

5	4	9	3	3
4	3	5	6	0
2	2	1	1	4
2	1	3	5	8

Layer 4

5	4	9	3	3
4	3	5	6	0
2	2	1	1	4
2	1	3	5	8

Layer 5



Observe that the top-left cell is covered in each of these layers. This cake has volume $16 + 8 + 8 + 3 + 1 = 36$ which is the maximum possible for this case.

Subtasks & Constraints

For all subtasks, $1 \leq R, C \leq 1\,000$, and $0 \leq H_{ij} \leq 1\,000\,000$ for all $1 \leq i \leq R$ and $1 \leq j \leq C$.

- For Subtask 1 (20 points), $H_{ij} \leq 1$ for all i and j.
- For Subtask 2 (25 points), $H_{ij} \leq 25$ for all i and j.
- For Subtask 3 (25 points), $R, C \leq 80$.
- For Subtask 4 (30 points), no further contraints apply.

Problem 3
Optimal Delivery

Time and Memory Limits: 4 seconds, 256 MB

You have just moved to a new city, and started a job as a courier with Optimal Delivery. The city is laid out on a grid of squares with R rows and C columns, with squares labelled from $(0, 0)$ in the north-west corner to $(R - 1, C - 1)$ in the south-east corner. It is only possible to travel between squares that are adjacent either horizontally or vertically.

Being new here, you haven't quite figured out the travel times between squares. A more experienced colleague tells you that all the travel times were chosen uniformly at random between 0 and 1 minute when the city was built, and have since been kept constant for tax purposes.

In order to avoid embarrassing your employer, you must first complete some mandatory training. You are to propose up to Q routes (one at a time), each between any two squares of your choice, and you will be told **how much longer** your route takes than the fastest route between those two squares.

Once you have completed your training, you will be asked to deliver a series of M parcels. For each parcel, you will be told the pickup square and the destination square. For each parcel, you may assume that the pickup square was chosen uniformly at random among all $R*C$ squares in the city and that the destination square was then chosen uniformly at random among the remaining $R * C - 1$ squares in the city. Then, you must propose a route to deliver the parcel as fast as you can. You will be given points based on **how much longer** your route takes than the fastest route between those two squares.

Be warned though, Optimal Delivery has high expectations of its new recruits, and will immediately fire you if you propose a route that passes through the same square more than once, or a route that passes outside the city grid, both during training and on the job.

Input / Output

This task has no input or output files. Instead your solution must interact with the functions the header file `"courier.h"`. The provided functions are described in detail in the next section.
***Do not* output *anything* to `stdout`, or you will receive 0 points for the test case.**

Module

Your program must interact with functions in `"courier.h"` as follows:

- Do *not* implement a `main` function — this is provided by the module. Instead, implement the function

  ```
  void init(int R, int C, int M, int Q)
  ```

  This function will be called *exactly once* after your program is launched. From this function, you are permitted to make up to Q calls to the function

  ```
  double query(int rStart, int cStart, int length, const char* path)
  ```

  where:

  - `rStart` and `cStart` are the row and column of the square in which your proposed route starts,
  - `length` is the length of the route (the number of times you move between squares), and
  - `path` is a string consisting only of characters `N`, `E`, `S`, and `W`, representing north, east, south, or west movements, respectively.

The `query` function will return how much longer, in minutes, your proposed route is than the fastest one. Each call to the `query` function takes time linearly proportional to `length`[1].

- You must also implement the function

  `void solve(int rStart, int cStart, int rEnd, int cEnd, int* length, char* path)`

  After your `init` function returns, this function will be called M times, representing the M deliveries you must make.

  - `rStart` and `cStart` are the row and column of the square in which you must pick up the parcel,
  - and `rEnd` and `cEnd` are the row and column of the square to which the parcel must be delivered.

  Note: the squares (`rStart`, `cStart`) and (`rEnd`, `cEnd`) will not be the same.

  Note: For each call made to solve, we will guarantee that (`rStart`, `cStart`) was chosen uniformly at random among all $R * C$ squares in the grid and that (`rEnd`, `cEnd`) was then chosen uniformly at random among the $R * C - 1$ remaining squares in the grid.

  You must return a valid path between these given endpoints by setting `*length` $= L$, where L is the length of your proposed route, and filling the array `path` with exactly L characters representing the movements between squares required to reach the destination, in the same format as described above. L must *not* exceed $RC - 1$.

  Your solution will receive points based on how much longer your proposed routes take than the fastest ones. See the *Scoring* section below for further details.

Experimentation

In order to experiment with your code on your own machine, first **download** the provided `courier.h` and `courier.c` files **to the same folder as your code file** from the *Statement* tab of the *Delivery* problem, and add `#include "courier.h"` to the top of your code. A sample solution is also available to assist you. Compile your C solution with

```
gcc -O2 -Wall -static yourcode.c courier.c -o courier -lm
```

or, if you use C++,

```
g++ --std=c++11 -O2 -Wall -static yourcode.cpp courier.c -o courier
```

where `yourcode.c/cpp` is the name of your code file. Note that you should still use `courier.c` even if your solution is in C++. The compiled executable `courier` will read 4 whitespace-separated integers `R C M Q` from standard input, followed by $2R - 1$ lines describing the travel times, where even-numbered lines contain $C - 1$ whitespace-separated decimal numbers between 0 and 1 (inclusive), and odd-numbered lines contain C whitespace separated decimal numbers between 0 and 1 (inclusive). This should be followed by M lines of the form $R_{S_i}\ C_{S_i}\ R_{E_i}\ C_{E_i}$, where (R_{S_i}, C_{S_i}) is the starting square and (R_{E_i}, C_{E_i}) is the ending square for the ith call that the module will make to the `solve` function. Such an input file will look something like this:

[1]This is true for the module used on the judging machine when you submit your solution, however, the `query` function in the `courier.h` file provided to you for experimentation may be slower.

$$
\begin{array}{cccccccccc}
R & & C & & M & & Q & & & \\
 & h_{0,0} & & & & h_{0,1} & & h_{0,2} & \cdots & h_{0,C-2} \\
v_{0,0} & & & v_{0,1} & & & v_{0,2} & & \cdots & & & v_{0,C-1} \\
 & h_{1,0} & & & & h_{1,1} & & h_{1,2} & \cdots & h_{1,C-2} \\
v_{1,0} & & & v_{1,1} & & & v_{1,2} & & \cdots & & & v_{1,C-1} \\
 & & & & & & \vdots & & & & & \\
v_{R-2,0} & & & v_{R-2,1} & & & v_{R-2,2} & & \cdots & & & v_{R-2,C-1} \\
 & h_{R-1,0} & & & & h_{R-1,1} & & h_{R-1,2} & \cdots & h_{R-1,C-2} \\
R_{S_1} & & C_{S_1} & & R_{E_1} & & C_{E_1} & & & \\
 & & & & & & \vdots & & & \\
R_{S_M} & & C_{S_M} & & R_{E_M} & & C_{E_M} & & &
\end{array}
$$

where $h_{i,j}$ is the travel time between square (i, j) and square $(i, j + 1)$, and $v_{i,j}$ is the travel time between square (i, j) and square $(i + 1, j)$.

Note: the input as illustrated here contains extra whitespace for clarity. This is not required by the module, but will be safely ignored if provided.

The executable will print M lines to standard output, where the ith line contains a decimal number s_i, the score your program receives for query i. This will be followed by a line containing `Total score: S`, where S is the total score for the test case. See the *Scoring* section for more details.

Sample Input

```
3 4 1 2
    0.5     1.0     0.9
0.1     0.6     0.7     0.4
    0.0     1.0     0.3
0.3     0.2     0.4     0.6
    0.0     1.0     0.5

2 1 0 2
```

Sample Session

1. The module calls `init(3, 4, 1, 2)`.
2. You call `query(0, 0, 3, "EEE")`. The function returns 0.6 since the proposed route takes 2.4 minutes, while the fastest possible route `"SEEEN"` takes 1.8 minutes.
3. You call `query(1, 0, 2, "ES")`. This path takes 0.2 minutes and is the fastest route, so the function returns 0.
4. Your `init` function returns.
5. The module calls `solve(2, 1, 0, 2, &length, path)`, as specified in the input, asking you to find as quick a route as you can between squares $(2, 1)$ and $(0, 2)$.

   You set `*length = 3` and `path[0] = 'N', path[1] = 'E', path[2] = 'N'`. The module outputs `33` to standard output, the score for this query.
6. The module outputs `Total score: 33` to standard output, the total score for this test case.

Subtasks & Constraints

For all calls to `solve`, $0 \leq$ `rStart`, `rEnd` $< R$ and $0 \leq$ `cStart`, `cEnd` $< C$, and `length` and `path` will always be non-`NULL` pointers. Also note that for all cases within a subtask, the values of R, C, M and Q are fixed, as specified:

- For Subtask 1 (30 points), $R = 2$, $C = 10$, $M = 1000$, $Q = 5000$.
- For Subtask 2 (15 points), $R = C = 10$, $M = 1000$, $Q = 25000$.
- For Subtask 3 (15 points), $R = C = 50$, $M = 1000$, $Q = 25000$.
- For Subtask 4 (20 points), $R = C = 50$, $M = 1000$, $Q = 4999$, and all travel times are either 0 or 1.
- For Subtask 5 (20 points), $R = C = 50$, $M = 1000$, $Q = 5000$.

Scoring

Your score for this problem will be the **maximum** score among all your submissions to the problem. For each submission, your score for each subtask will be the **minimum** score among all test cases in that subtask. Your score for each test case will be

$$\max\left(0, \frac{\max\left(t_M, t_F + 10^{-6}\right) - t}{\max\left(t_M, t_F + 10^{-6}\right) - t_F} \times 100\right)$$

where t, t_F, and t_M are the sums of all t_i, t_{Fi}, and t_{Mi}, respectively. For the ith call to `solve`:

- t_i is the time taken by your proposed route,
- t_{Fi} is the time taken by the fastest possible route, and
- t_{Mi} is the *mean* of the time taken by the route which travels first in the north-south direction to the destination's grid row, then in the east-west direction to the destination; and the route which travels first in the east-west direction to the destination's grid column, then in the north-south direction to the destination.

If you propose a route that results in you being fired, you will receive 0 points for the test case.
If you output anything to `stdout`, you will receive 0 points for the test case.