French-Australian Regional Informatics Olympiad

Sunday 8th May

2005

Duration: 4 hours

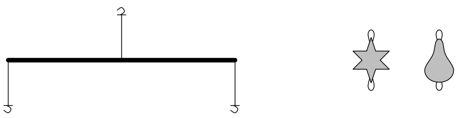
3 Questions

All questions should be attempted

Problem 1
Mobile Construction Set

Time and Memory Limits: 10 seconds, 10 Mb

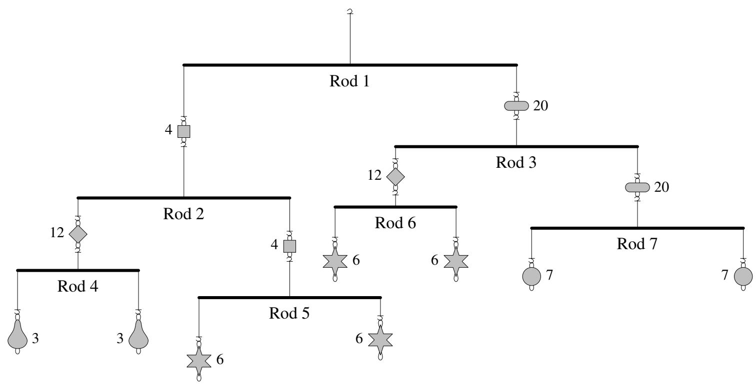
Your younger sister just got a mobile construction set for her birthday. The set consists of two different kinds of elements: rods and decorations, as illustrated below.



Each rod has three hooks attached via pieces of string. In the middle of each rod, a string rises upwards with a hook at the top. This allows the entire rod to be hung beneath some other object (either the ceiling or a higher decoration). One and only one rod must attach the mobile to the ceiling. At the ends of each rod, strings fall down with hooks at the bottom. This allows a decoration to be hung beneath the rod at each end.

There are many types of decorations, with differing shapes and weights. Each decoration has two rings attached to it, one at the top and one at the bottom. The top ring can be used to hang the decoration on a hook beneath the left or right end of a rod. The bottom ring can be used to hang another rod beneath the decoration.

When building a mobile with this set, no hook may be left free except for the very top hook that hangs the entire mobile from the ceiling. Every other hook must be attached to a decoration. Decorations are not so constrained; some decorations may have other rods hanging beneath them, whereas other decorations may have nothing attached to their bottom ring. An example of a completed mobile is illustrated below. The rods are numbered from 1 to 7, and the weight of each decoration is written beside it.



Your younger sister has finished making a beautiful mobile with her set, but she is complaining that it doesn't work! Indeed, the mobile is far from being well balanced: what is hanging on one side of a rod is sometimes much heavier than what is hanging on the other side.

You have offered your help, but your sister gets upset when you explain that you will have to move some things around. She doesn't want you to change anything in her beautiful construction — it's perfect the way it is, you just have to make it work! You manage to convince her that you need to change at least something to improve the balance, but she will only allow you to make a single change to her creation.

Your task is to make the entire mobile as balanced as possible, by making only one modification to it. This modification must involve detaching a rod from the decoration under which it is hanging, and then reattaching it beneath some other decoration. You don't need to worry about whether different parts of the mobile might bump into each other, since you can always fix this afterwards by changing the lengths of some strings. Making the mobile balanced is your only concern.

To measure how balanced a rod is, you must calculate the absolute value of the difference between the total weights hanging from each end (so a lower number is better). To measure how balanced the entire mobile is, you must add up these differences for every rod. Your goal is to reduce this total to its smallest possible value by making at most one change. Note that you are not required to make a change; you may choose to move nothing at all.

Note also that when moving a rod, you may not attach the rod to a decoration that already has another rod hanging from it. Rods, strings and hooks are so light that they can be considered as having no weight at all.

Constraints

- $1 \leq N \leq 2000$, where N is the number of rods;
- $1 \leq W \leq 1000$, where W is the weight of a decoration.

Furthermore, for 30% of the available marks, $N \leq 200$.

Input

The first line of input contains one integer: N, the number of rods used in the mobile. The rods are numbered from 1 to N, with rod 1 being attached to the ceiling.

Each of the following N lines describes a rod. These rods are given in order from 1 to N (so that the second input line describes rod 1, the third input line describes rod 2 and so on). Each of these lines is of the form W_1 R_1 W_2 R_2, where W_1 and W_2 are the weights of the decorations attached directly beneath the left and right ends of the rod, and R_1 and R_2 are the numbers of the rods hanging beneath these decorations (where rods R_1 and R_2 hang beneath the decorations of weight W_1 and W_2 respectively). If a decoration has no rod hanging beneath it, the corresponding rod number will be zero.

Output

The output should contain one line, with a single integer: the minimum sum of weight differences that you can get by moving at most one rod of the mobile.

Sample Input

The following input scenario describes the large mobile illustrated earlier.

```
7
4 2 20 3
12 4 4 5
12 6 20 7
3 0 3 0
6 0 6 0
6 0 6 0
7 0 7 0
```

Sample Output

```
42
```

In the input scenario, the weight differences for rods $1, \ldots, 7$ are 40, 2, 10, 0, 0, 0 and 0 respectively, coming to a total sum of 52. By moving rod 7 so that it hangs beneath one of the decorations attached to rod 5, these weight differences become 12, 12, 4, 0, 14, 0 and 0, coming to a total sum of 42. No smaller total sum can be obtained without moving more than one rod, and so 42 is the final answer.

Scoring

The score for each input scenario will be 100% if the correct answer is output, and 0% otherwise.

Problem 2
Monkey Tour

Time and Memory Limits: 1 second, 64 Mb

Trevor the monkey has been placed into an artificial animal reserve. It's rather obvious that the forest is not natural: all of the trees have been planted in a perfect grid formation, with each tree separated by 1 metre from its neighbour! Trees in this forest are so close to each other that no large branches can grow, and as a consequence Trevor has no peaceful place to rest. He solves this problem by building a certain number of nests, N, inside some of the trees.

The management of the reserve have defined Trevor's territory to be a square area of side S metres located well inside the reserve. All of Trevor's nests lie within this square or on its boundary. Trevor is allowed to move outside his territory, but the reserve is so large that even if he does this there is no way he can reach the border.

Since all the branches are very small, it is quite hazardous for a big monkey like Trevor to climb from tree to tree. Because of this, Trevor will only move from tree to tree by swinging on vines, as Tarzan used to teach him when he was young. Vines can always be found wherever they are neeeded in this forest, and so this method of transport is quick and efficient. There are however some restrictions:

- To swing between two trees, there cannot be any other tree on a straight line between them (otherwise, smashed monkey).
- Vines are only long enough to travel a distance L or less in a single swing.

Swinging on vines is quite exhausting, even for a trained monkey, and so Trevor would like to minimise his fatigue as he travels between his nests. To move between two nests, he uses vines to swing from tree to tree, possibly having little rests at other nests along the way. Since these rests are quite refreshing, Trevor does not measure his fatigue by the total number of swings. Instead he measures his fatigue by the largest number of consecutive swings taken without any rests in between.

Trevor is quite lazy and not that clever — he is a real monkey. He'd like you to help him find a group of at least $N/2$ nests that he can travel between, so that the greatest fatigue that he must endure between any of these $N/2$ nests is as small as possible. If N is odd then $N/2$ should be rounded up to the integer above.

Constraints

- $1 \leq L \leq 15$, where L is the maximum distance for a single swing;
- $2 \leq S \leq 200$, where S is the side length of Trevor's territory;
- $2 \leq N \leq 1000$, where N is the number of nests that Trevor has built;
- $0 \leq X_i < S$ and $0 \leq Y_i < S$, where (X_i, Y_i) are the co-ordinates of the ith nest.

Furthermore, for 30% of the available marks you are guaranteed that $L \leq 3$, $S \leq 100$ and $N \leq 65$.

Input

The first line of input will contain the single integer L, giving the maximum distance that can be travelled in a single swing (measured in metres). The second line of input will contain the single integer S, giving the side length of the square territory (again measured in metres). The third line of input will contain the single integer N, giving the total number of nests.

Following this will be N lines, each containing two integers X_i and Y_i separated by a space, giving the x-coordinate and y-coordinate of each nest. No two nests will occupy the same location.

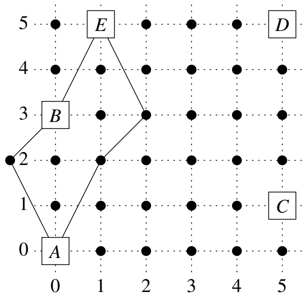
Output

Output should consist of a single line containing a single integer: the minimum fatigue that Trevor must endure in order to be able to travel anywhere within some group of at least $N/2$ nests.

Sample Input

```
3
6
5
0 0
0 3
5 1
5 5
1 5
```

The input scenario above describes the forest in the illustration below. Trees are marked with black circles, and the five nests are marked with the letters A, B, C, D and E. Note that in this example we can swing a maximum distance of $L = 3$ metres.



Consider nests A and B. Although these are three metres away, Trevor cannot swing directly between them since there are other trees in the way. He therefore needs at least two swings. For example, he could swing from $A = (0, 0)$ to $(-1, 2)$ and from there to $B = (0, 3)$. This route is marked with a solid line on the map. Note that this route goes outside Trevor's territory (which is perfectly allowable).

To travel between nests A and E, Trevor needs at least three swings; again he cannot take a more direct route since there are other trees in the way. One possible route from A to E might be from $A = (0, 0)$ to $(1, 2)$, then to $(2, 3)$ and finally to $E = (1, 5)$.

Note however that Trevor can travel from A to E with a fatigue of only 2. He does this by travelling from A to B with two swings, then resting in nest B, and finally travelling on from B to E with one additional swing. Since at most two consecutive swings are taken without a rest between them, the total fatigue for this new journey from A to E is 2.

Sample Output

```
2
```

Recall that Trevor is seeking a group of three nests ($N/2$ rounded up) that he can travel between. If he chooses nests A, B and E, we see that he can reach any nest from any other nest within this group with a fatigue of at most 2. There is no other group of three nests that gives a better result (though there are many others that give the same result), and so 2 is the final answer.

Scoring

The score for each input scenario will be 100% if the correct answer is output, and 0% otherwise.

Problem 3
River

Time and Memory Limits: 1 second, 10 Mb

A lush, beautiful and uninhabited island has been discovered in a remote patch of the Pacific Ocean. Several foreigners are planning to migrate to this little piece of paradise, and you have found yourself in charge of planning the main town.

As it happens, most of the migrants are French and Australian. Unfortunately none of the Australians ever learned French, and none of the French can understand the thick Australian accents.

You have therefore decided to split the town into two sectors. A wide river flows through the centre of the island, and so you plan to form a French sector on one side of the river and an Australian sector on the other. In the middle of the river, a few floating restaurants will open up so that the different nationalities can meet and speak the universal language of coffee.

You must be very careful about how you build this town. The residents wish to keep the sectors as equal as possible, so that one nationality does not dominate the other. They keep careful track of how much area has been developed on each side of the river, and they tax you (the town planner) according to how large the difference is.

Specifically, each time you construct a new building, the residents measure the total area of buildings on the French side and the total area of buildings on the Australian side (in square metres). They then calculate the difference between these two areas, and tax you by this same amount in drachmas (the only form of currency that both countries can agree upon).

The residents are also quite fussy about which buildings they want constructed. You have been given an exact list of which buildings must be built, though you are allowed to build them in any order and on any side of the river. Your task is to choose when and where to construct these buildings so that you pay less tax in total.

Example

Suppose that you are asked for three buildings of areas 2, 3 and 4. The tables below illustrate two different ways in which you might construct these buildings.

New Building	**Area (Aus.)**	**Area (Fr.)**	**Tax**
Area 2, French side	0	2	2
Area 3, French side	0	5	5
Area 4, Australian side	4	5	1
Total tax			**8**

New Building	**Area (Aus.)**	**Area (Fr.)**	**Tax**
Area 3, Australian side	3	0	3
Area 4, French side	3	4	1
Area 2, Australian side	5	4	1
Total tax			**5**

The second method is better than the first, since it gives a lower total tax of 5 drachmas.

Constraints

- $1 \leq N \leq 100$, where N is the number of buildings that are required;
- $1 \leq A \leq 100\,000$, where A is the area of a building (measured in square metres).

Furthermore, for 30% of the available marks, no building will have an area greater than 100.

Input

Your program must read from standard input. The first line will be a single integer N, denoting the total number of buildings that are required.

Following this will be N lines each describing a single building. Each of these lines will contain a single integer representing the area of the building (in square metres). Note that there may be several buildings each having the same area.

Output

Your program must write the best solution it can find to standard output. Your output should begin with N lines describing the N buildings in the order in which you construct them. Each of these lines should have the form A S, where A is the area of the building and S denotes which side of the river it is built on. The area A must be an integer (measured in square metres), and S must be a single lower-case letter "`a`" or "`f`", denoting the Australian or French side respectively.

After the buildings have been written, your program must write one additional line of output. This line must contain a single integer representing the total tax paid (in drachmas).

Sample Input

```
3
2
3
4
```

Sample Output

```
3 a
4 f
2 a
5
```

Scoring

There is no particular "best solution" that you are required to achieve. Instead, your score will be determined relative to the other contestants whom you are competing against (as well as the judges' solution).

For each input scenario, the contestant who achieves the least total tax will be identified. Suppose that this contestant obtains a total tax of T. Furthermore, let the average building area for the input scenario be M (so that M is the sum of all building areas divided by N). Your score for this input scenario will then be:

- 100% if your program finds a solution with this same least total tax T;
- 10% if your program finds a solution whose total tax is $T + M$ or greater;
- 0% if your program generates an incorrect solution (e.g., you miss a building, construct a building more than once, or incorrectly calculate the total tax);
- otherwise determined by a linear scale according to your total tax, with the 100% and 10% marks on the scale corresponding to the solutions described above.

For example, consider an input scenario for which the average building area is $M = 90$. If the best solution found by any contestant (or the judges) has a total tax of 400, then the scoring scale for a *correct* solution would be as follows:

Total tax	400	420	440	460	480	490	500	510
Score	100%	80%	60%	40%	20%	10%	10%	10%